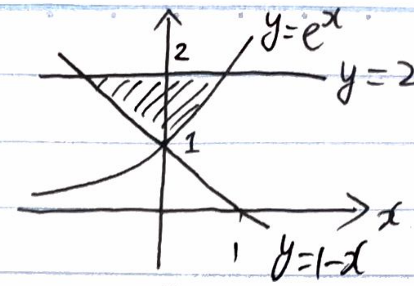
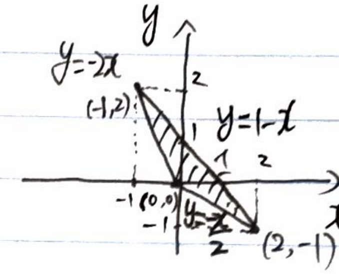
§15.3



10. $\int_1^2\int_{1-y}^{\ln y} dx\,dy = \int_1^2 x\Big|_{1-y}^{\ln y} dy$
$=\int_1^2 \ln y - 1 + y\, dy = \left(y\ln y - 2y + \frac{y^2}{2}\right)\Big|_1^2$
$= 2\ln 2 - \frac{1}{2}$



17. $\int_{-1}^{0}\int_{-2x}^{1-x} dy\,dx + \int_0^2\int_{-x/2}^{1-x} dy\,dx$
$= \int_{-1}^{0} 1+x\, dx + \int_0^2 1 - x/2\, dx$
$= (x + x^2/2)\Big|_{-1}^{0} + (x - x^2/4)\Big|_0^2 = 3/2$

20. average value over the square $= \int_0^1\int_0^1 xy\,dy\,dx = \int_0^1 \frac{xy^2}{2}\Big|_0^1 dx = \int_0^1 \frac{x}{2}dx = \frac{1}{4} = 0.25$
average value over the quarter circle $= \frac{1}{(\pi/4)}\int_0^1\int_0^{\sqrt{1-x^2}} yx\,dy\,dx = \frac{4}{\pi}\int_0^1 \frac{xy^2}{2}\Big|_0^{\sqrt{1-x^2}} dx$
$= \frac{2}{\pi}\int_0^1 (x - x^3)\,dx = \frac{2}{\pi}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2\pi} \approx 0.159$
The average value over the square is larger.

§15.4

19. $\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy = \int_0^{\pi/2}\int_0^{\ln 2} re^r\,dr\,d\theta = \int_0^{\pi/2}(2\ln 2 - 1)\,d\theta = \frac{\pi}{2}(2\ln 2 - 1)$

30. $A = \int_0^{2\pi}\int_0^{4\theta/3} r\,dr\,d\theta = \frac{8}{9}\int_0^{2\pi}\theta^2 d\theta = \frac{64\pi^3}{27}$

44. $A = \int_\alpha^\beta\int_0^{f(\theta)} r\,dr\,d\theta = \int_\alpha^\beta \left(\frac{r^2}{2}\right)\Big|_0^{f(\theta)} d\theta = \frac{1}{2}\int_\alpha^\beta f^2(\theta)\,d\theta = \int_\alpha^\beta \frac{1}{2}r^2 d\theta,\ r = f(\theta)$

§15.5

17. $\int_0^\pi\int_0^\pi\int_0^\pi \cos(u+v+w)\,du\,dv\,dw = \int_0^\pi\int_0^\pi \sin(w+v+\pi) - \sin(w+v)\,dv\,dw$
$= \int_0^\pi \left(-\cos(w+2\pi) - \cos(w+\pi)\right) + \left(\cos(w+\pi) - \cos w\right) dw$
$= \left[-\sin(w+2\pi) + \sin(w+\pi) - \sin w + \sin(w+\pi)\right]\Big|_0^\pi = 0$

25. $V = \int_0^4\int_0^{\sqrt{4-x}}\int_0^{2-y} dz\,dy\,dx = \int_0^4\int_0^{\sqrt{4-x}} (2-y)\,dy\,dx = \int_0^4 2\sqrt{4-x} - \left(\frac{4-x}{2}\right) dx$
$= \left[-\frac{4}{3}(4-x)^{\frac{3}{2}} + \frac{1}{4}(4-x)^2\right]\Big|_0^4 = \frac{20}{3}$

29. $V = 8\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2}} dz\,dy\,dx = 8\int_0^1\int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\,dy\,dx = 8\int_0^1 (1-x^2)\,dx = \frac{16}{3}$

46. The volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{4}{3}abc\pi$, so that $\frac{4}{3}\times 1\times 2\times c\pi = 8\pi \Rightarrow c = 3$